

The bang-bang funnel controller: An experimental verification

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We adjust the newly developed bang-bang funnel controller such that it is more applicable for real world scenarios. The main idea is to introduce a third “neutral” input value to account for the situation when the error is already small enough and no control action is necessary. We present experimental results to illustrate the effectiveness of our new approach.

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1 Introduction and controller design

The recently introduced bang-bang funnel controller (BBFC) [3] is able to achieve reference signal tracking of an uncertain plant of which only is known that the relative degree is either one or two. This theoretical result involves certain feasibility assumptions and it is not clear in general whether these are satisfiable and realistic in real world scenarios. We therefore want to study the applicability of the BBFC to an real experimental setup. We apply the BBFC to a stiffly coupled rotary machine as shown in Figure 1 on which the continuous funnel controller was already applied successfully [2].

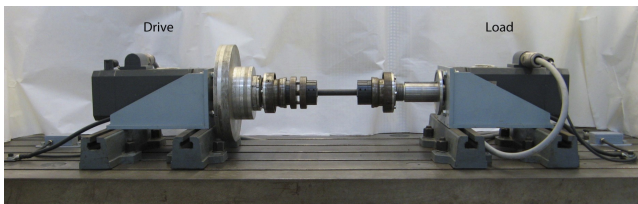


Fig. 1 Experimental setup.

Applying the BBFC in its pure form would result in high stress on the machine because of switching between applying the maximum positive moment and the maximal negative momentum. This mechanical stress can be relaxed by introducing the third possibility of doing nothing (i.e. applying zero momentum), hence we consider a feedback loop as illustrated in the left of Figure 2.

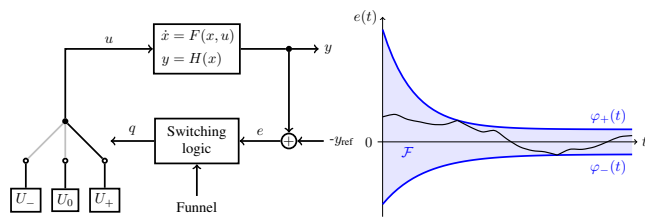


Fig. 2 Overall feedback configuration and the funnel.

This makes it necessary to define a new switching logic. However, the new switching logic is very similar to the one in [3]; in particular, all the theoretical results remain valid.

The control objective is keeping the error $e := y - y_{ref}$ between the output y of the system and the reference signal y_{ref}

within the *funnel*

$$\mathcal{F} := \{ (t, e) \mid \varphi_-(t) \leq e \leq \varphi_+(t) \} \quad (1)$$

where $\varphi_+ : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{>0}$ and $\varphi_- : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{<0}$ are pre-specified (time-varying) error bounds, see Figure 2. This goal should be achieved via error feedback and the application of a bang-bang control with input values U_+ , U_- and the neutral input value U_0 . The decision which input value is used at a given time t is made by a switching logic. In our setup the switching logic depends on the error e and its derivative \dot{e} . The output of the switching logic is the decision variable $q : \mathbb{R}_{\geq 0} \rightarrow \{-1, 0, 1\}$ and the control law is then simply

$$u(t) = \begin{cases} U_-, & \text{if } q(t) = -1, \\ U_0, & \text{if } q(t) = 0, \\ U_+, & \text{if } q(t) = +1. \end{cases}$$

The switching logic will be defined with the help of the following basic switching rule

$$\mathcal{S}(e, e^+, e_0, e^-, q^{old}) := \begin{cases} -1, & \text{if } e = e^+ \vee (q^{old} = -1 \wedge e > 0), \\ 0, & \text{if } e = 0 \vee (q^{old} = 0 \wedge e^- < e < e^+), \\ +1, & \text{if } e = e^- \vee (q^{old} = +1 \wedge e < 0). \end{cases}$$

Furthermore, the switching logic contains the internal logical variable q_1 defined via $q_1(0-) = q_1^0 \in \{-1, 0, 1\}$ and

$$q_1(t) = \mathcal{S}(e(t), \varphi_+(t) - \varepsilon_+, 0, \varphi_-(t) + \varepsilon_-, q_1(t-)),$$

where $\varepsilon_{\pm} > 0$ is the safety distance as introduced in [3]. The definition for q_1 and its interpretation is shown in Figure 3.

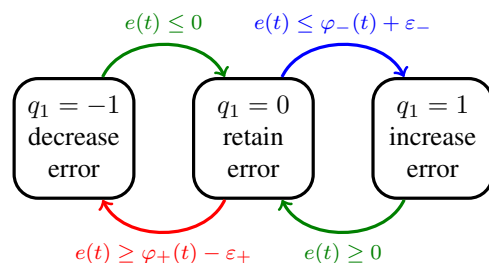


Fig. 3 Switching logic for q_1 .

The switching logic for q itself is given in terms of the error derivative \dot{e} together with its corresponding funnel \mathcal{F}^d analogously given as in (1) with boundaries φ_{\pm}^d . This funnel must

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be feasible in the sense that $\varphi_+^d > \dot{\varphi}_-$ and $\varphi_-^d < \dot{\varphi}_+$ uniformly on $\mathbb{R}_{\geq 0}$. Furthermore, we need some more (time-varying) switching triggers, namely $\varphi_{+0}^d, \varphi_{-0}^d, \psi_+$ and ψ_- with $\varphi_+^d > \varphi_{+0}^d > \dot{\varphi}_-, \varphi_-^d < \varphi_{-0}^d < \dot{\varphi}_+$ and $\varphi_-^d < \psi_- < 0 < \psi_+ < \varphi_+^d$ uniformly on $\mathbb{R}_{\geq 0}$. Then q is given via $q(0-) = q^0 \in \{-1, 0, 1\}$ and

$$\begin{aligned} q(t) &= \mathcal{S}(\dot{e}(t), \dot{\varphi}_+(t), \varphi_{-0}^d(t), \varphi_-^d(t) + \varepsilon_-^d, q(t-)), \\ &\quad \text{if } q_1(t) = -1, \\ q(t) &= \mathcal{S}(\dot{e}(t), \psi_+(t), 0, \psi_-(t), q(t-)), \\ &\quad \text{if } q_1(t) = 0 \text{ and} \\ q(t) &= \mathcal{S}(\dot{e}(t), \varphi_+^d(t) - \varepsilon_+^d, \varphi_{+0}^d(t), \dot{\varphi}_-(t), q(t-)), \\ &\quad \text{if } q_1(t) = 1. \end{aligned}$$

Here $\varepsilon_{\pm}^d > 0$ is a safety distance which is theoretically not necessary but advisable in practical application.

2 Experimental results

A mathematical model of the rotary machine is given by the differential equations

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \gamma \end{bmatrix} (u(t) + u_L(t) - (Tx_2)(t)), \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t), \end{aligned}$$

where x_1 denotes the angle of the rotary machine, $x_2 = \dot{x}_1$ is the angular velocity, u_L is the unknown load torque and $T : \mathcal{C}(\mathbb{R}_{\geq 0} \rightarrow \mathbb{R}) \rightarrow \mathcal{L}_{\text{loc}}^{\infty}(\mathbb{R}_p \rightarrow \mathbb{R})$ is a friction operator, for details see [1]. As funnel boundaries we choose exponentially decaying functions and the switching triggers are $\varphi_{+0}^d = \psi_+ = \dot{\varphi}_- + \delta$ and $\varphi_{-0}^d = \psi_- = \dot{\varphi}_+ - \delta$, where $\delta > 0$. The input values are

$$U_+ = 22 \text{ Nm}, U_0 = 0 \text{ Nm}, U_- = -22 \text{ Nm}.$$

The overall experimental results on the interval $[0 \text{ s}, 40 \text{ s}]$ together with the load disturbance u_L is shown in Figure 4.

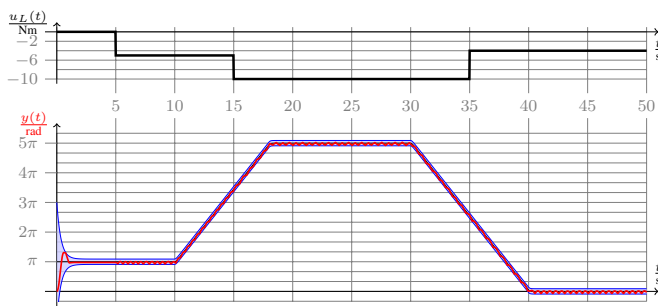


Fig. 4 Overall reference tracking performance, upper part: load disturbance u_L ; lower part: output y (red) follows reference signal y_{ref} (black) and the error remains within the funnel (blue).

The transient response is shown Figure 5 which is a zoom of Figure 4 to the interval $[0 \text{ s}, 4 \text{ s}]$, where also the derivative of the error and the results of the switching logic is shown.

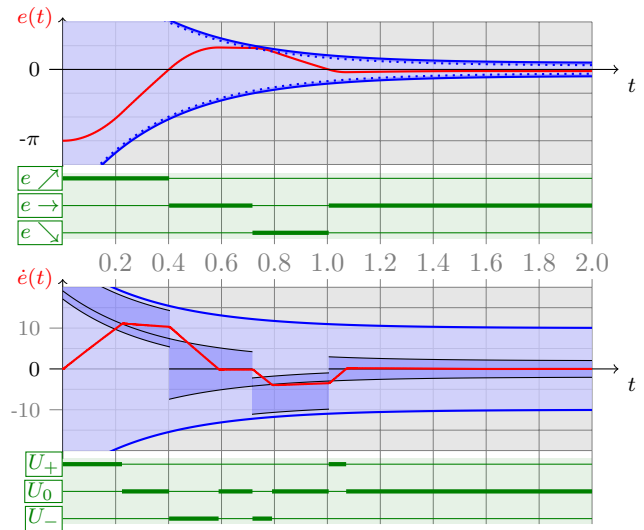


Fig. 5 Transient response without load disturbance.

Finally, another zoom of Figure 4 to the interval $[5 \text{ s}, 7 \text{ s}]$ is shown in Figure 6 to illustrate the effect of a non-zero disturbance load.

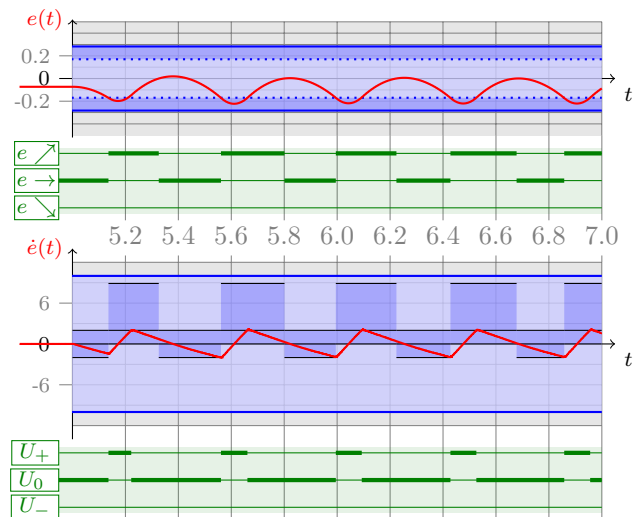


Fig. 6 Response in the presence of load disturbance.

References

- [1] Christoph M. Hackl. *Contributions to high-gain adaptive control in mechatronics*. PhD thesis, Fakultät für Elektrotechnik und Informationstechnik, TU München, 2012. In preperation.
- [2] Christoph M. Hackl, Norman Hopfe, Achim Ilchmann, Markus Mueller, and Stephan Trenn. Funnel control for systems with relative degree two. Submitted for publication, preprint available from the websites of the authors, 2010.
- [3] Daniel Liberzon and Stephan Trenn. The bang-bang funnel controller. In *Proc. 49th IEEE Conf. Decis. Control, Atlanta, USA*, pages 690–695, 2010.