

*2004 CCA/ISIC/CACSD*

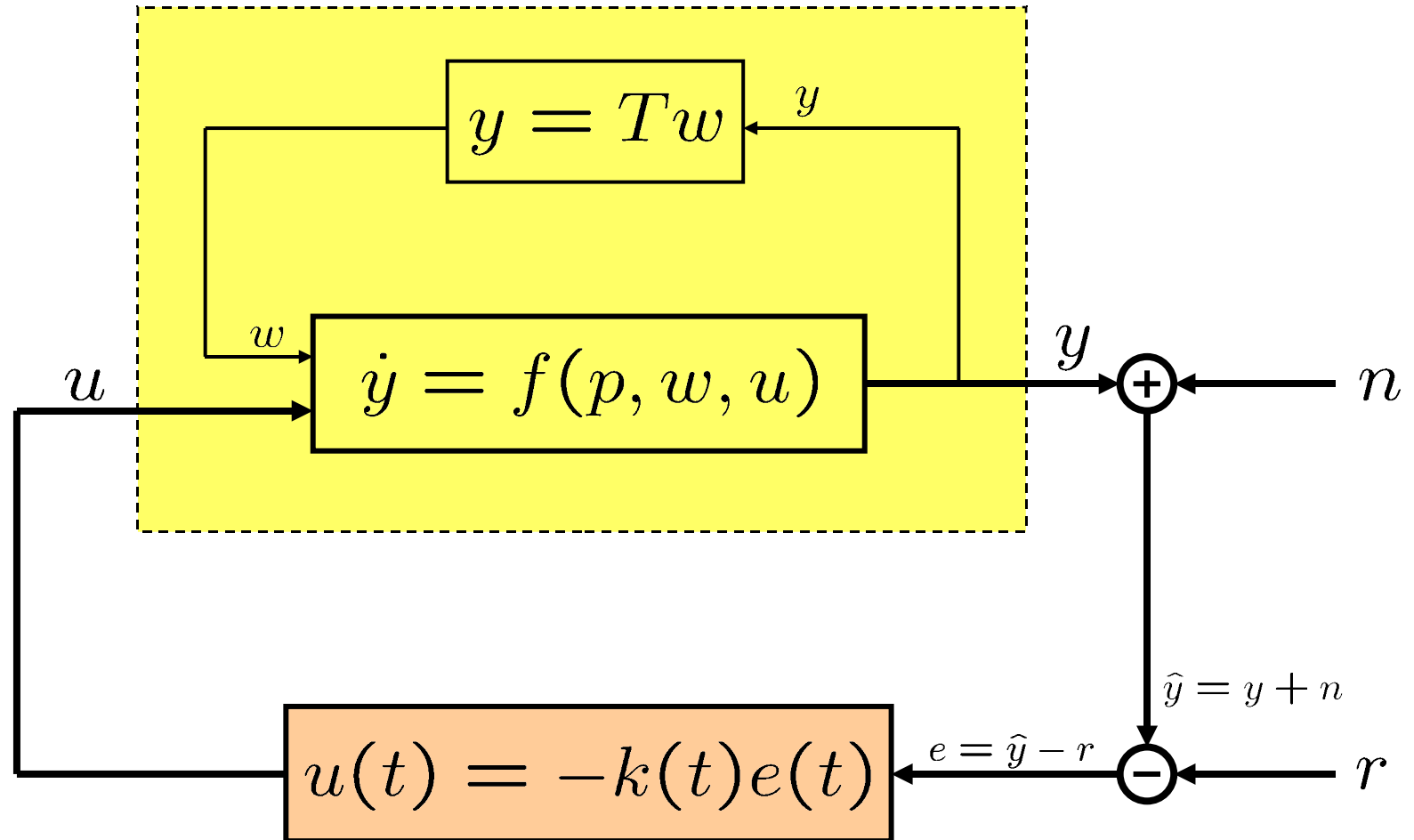
# Adaptive tracking within prescribed funnels

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**Aim:** output tracking of nonlinear system by proportional error feedback

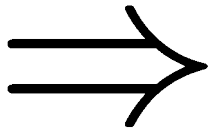


## Class of systems: Linear SISO example

$$\begin{aligned} \dot{y}(t) &= A_1 y(t) + A_2 z(t) + cb u(t) & y(0) &= y^0 \in \mathbb{R} \\ \dot{z}(t) &= A_3 y(t) + A_4 z(t) & z(0) &= z^0 \in \mathbb{R}^{n-1} \end{aligned}$$

$$cb > 0$$

$A_4$  Hurwitz



$u(t) = -k y(t)$  stabilises for  $k \gg 1$

## System class

$$\dot{y}(t) = f\left(p(t), (Ty)(t), u(t)\right) \quad y|_{[-h,0]} = y^0 \in C([-h, 0], \mathbb{R}^M)$$

1.  $p \in L^\infty(\mathbb{R}_{\geq 0}, \mathbb{R}^P)$

2.  $f \in C(\mathbb{R}^P \times \mathbb{R}^Q \times \mathbb{R}^M, \mathbb{R}^M)$

3.  $\forall$  compact  $\mathcal{C} \subseteq \mathbb{R}^P \times \mathbb{R}^Q \quad \forall (u_n) \in (\mathbb{R}^M \setminus \{0\})^{\mathbb{N}}$  :

$$\|u_n\| \rightarrow \infty \text{ as } n \rightarrow \infty \quad \Rightarrow \quad \lim_{n \rightarrow \infty} \min_{(v,w) \in \mathcal{C}} \langle u_n, f(v, w, u_n) \rangle / \|u_n\| = \infty$$

4. bounded, causal and locally Lipschitz operator

$$T : C([-h, \infty), \mathbb{R}^M) \rightarrow L_{\text{loc}}^\infty(\mathbb{R}_{\geq 0}, \mathbb{R}^Q)$$

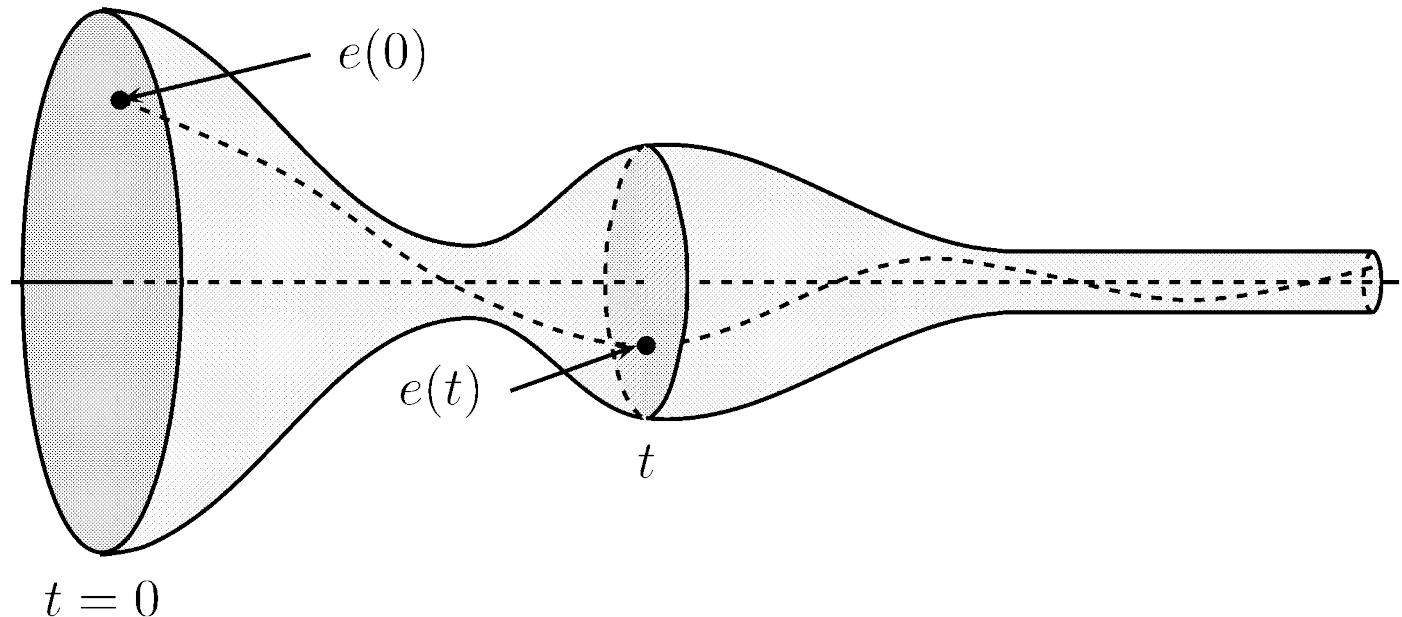
## Control objectives

$\lambda$ -tracking

prescribed  
transient  
behaviour

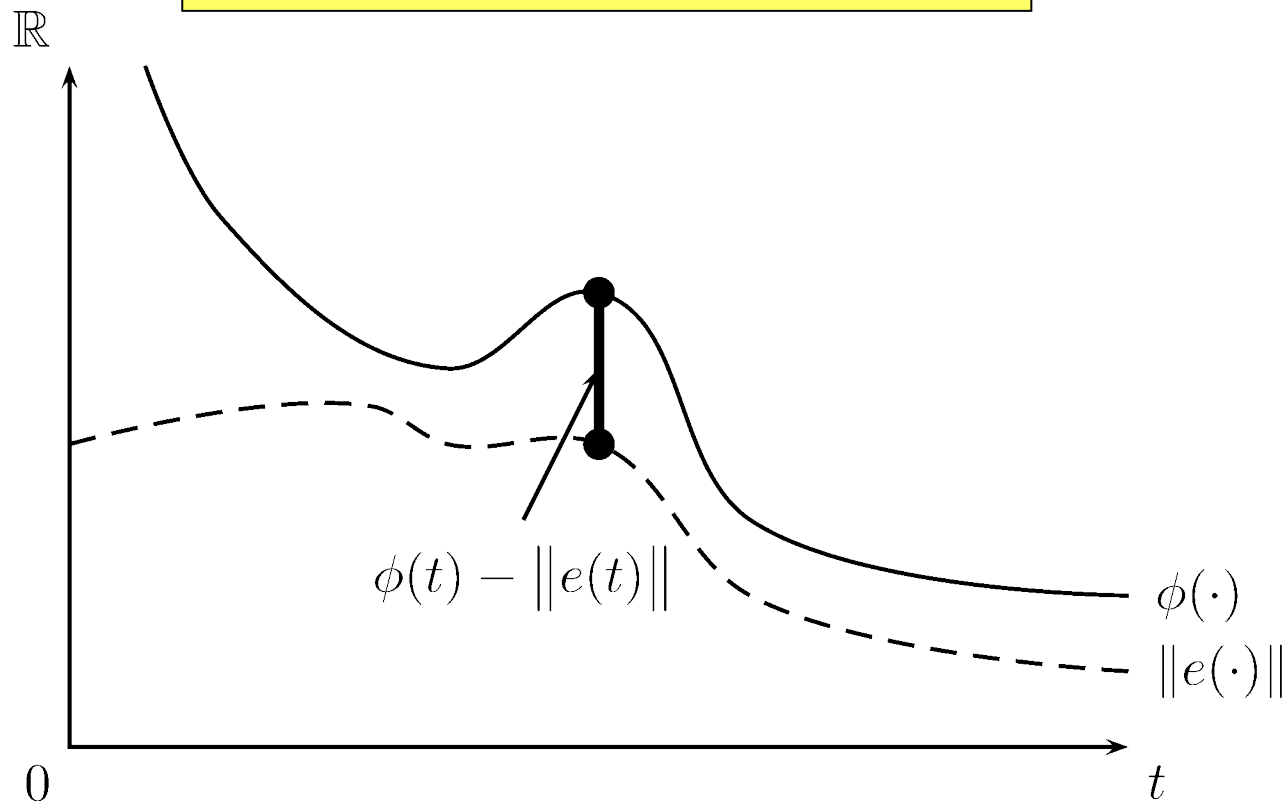
a *low*  
high-gain

funnel  $\mathcal{F}$



**Control input**  $u(t) = -k(t)e(t)$

$$k(t) = \frac{1}{\phi(t) - \|e(t)\|}$$

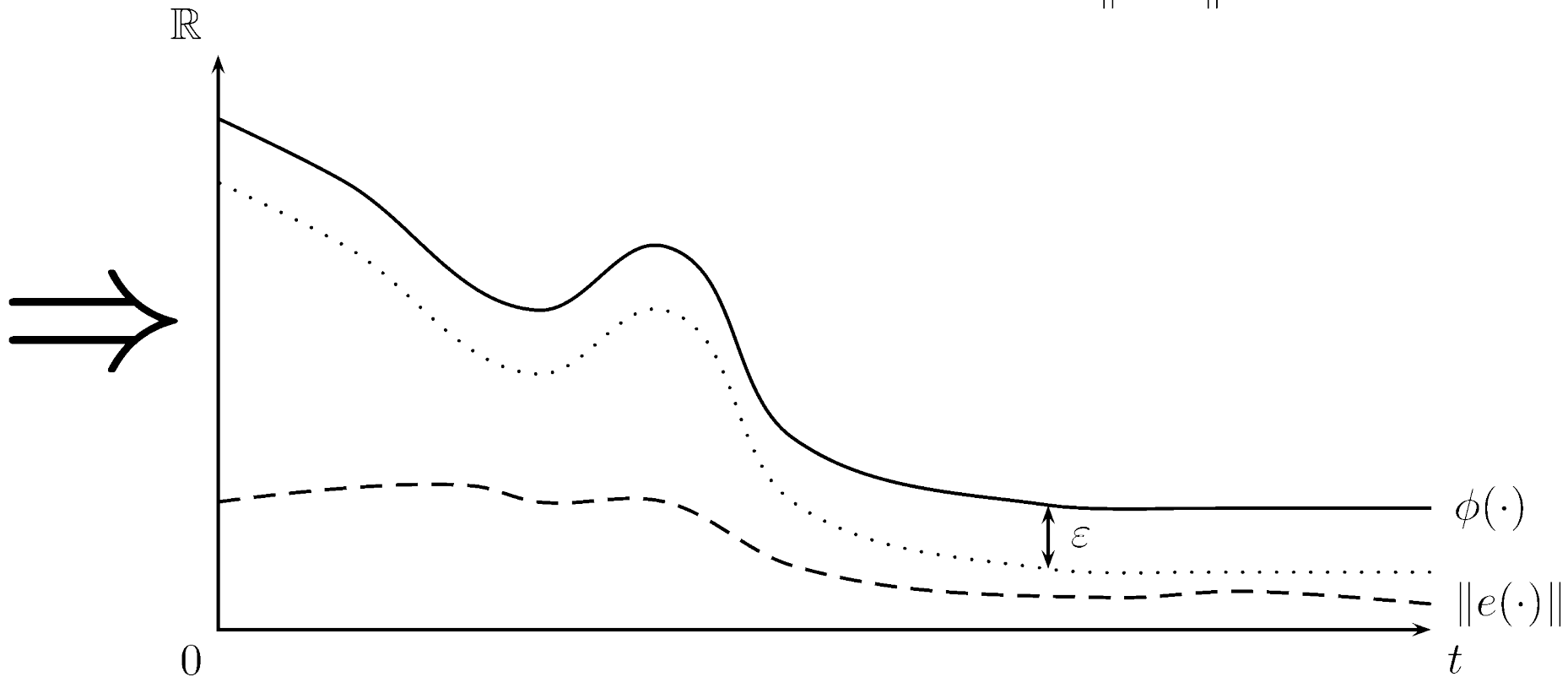


## Theorem (Linear example)

$$\dot{y}(t) = A_1 y(t) + A_2 z(t) + cb u(t) \quad y(0) = y^0 \in \mathbb{R}$$

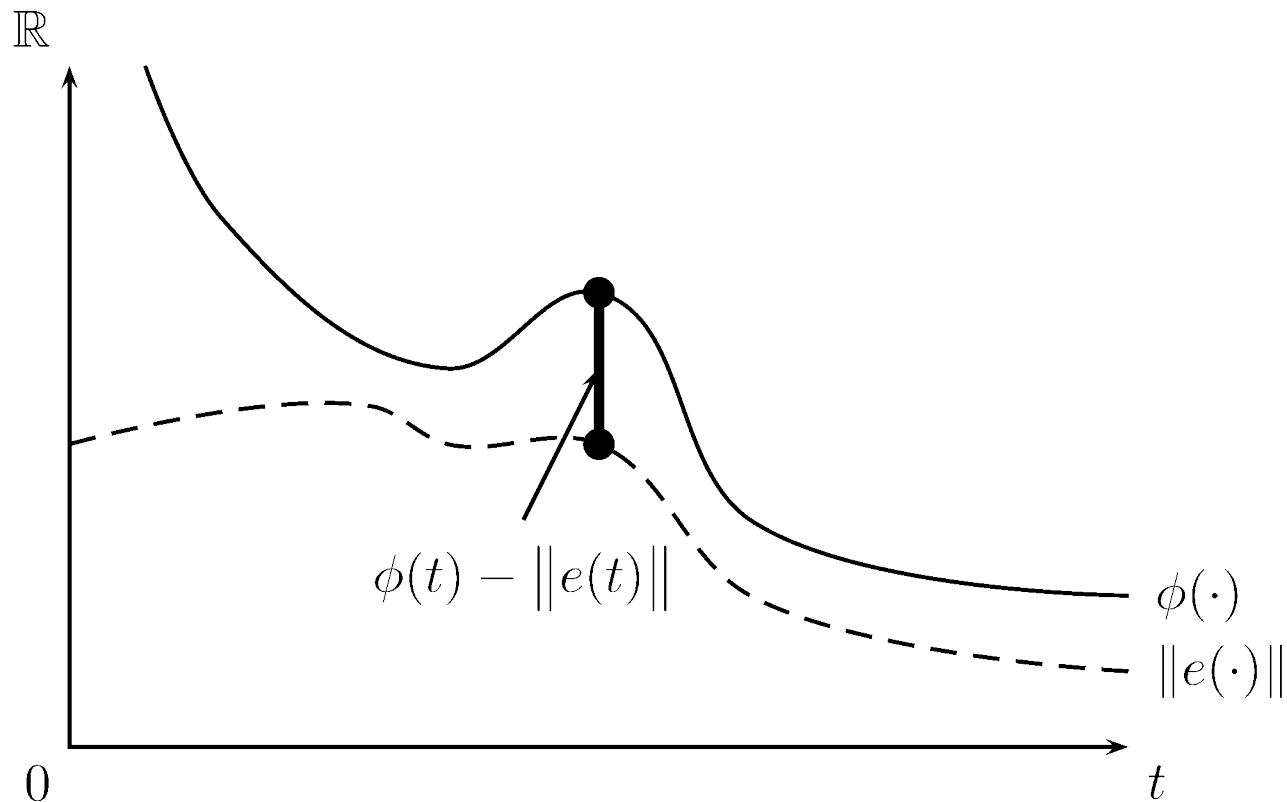
$$\dot{z}(t) = A_3 y(t) + A_4 z(t) \quad z(0) = z^0 \in \mathbb{R}^n$$

$$cb > 0 \quad A_4 \text{ Hurwitz} \quad u(t) = - \frac{1}{\phi(t) - \|e(t)\|} e(t)$$



**Control input**  $u(t) = -k(t)e(t)$

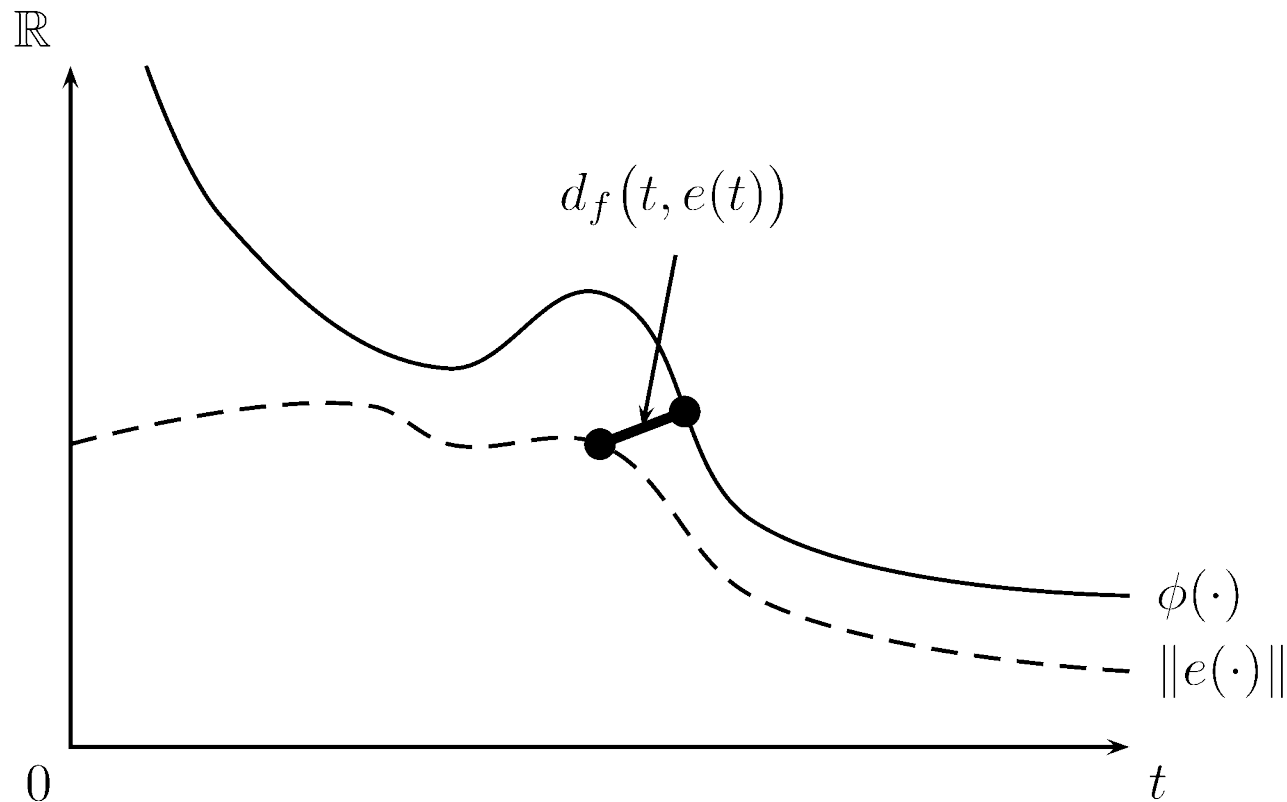
$$k(t) = \frac{1}{\phi(t) - \|e(t)\|} = K_{\mathcal{F}}(t, e(t))$$





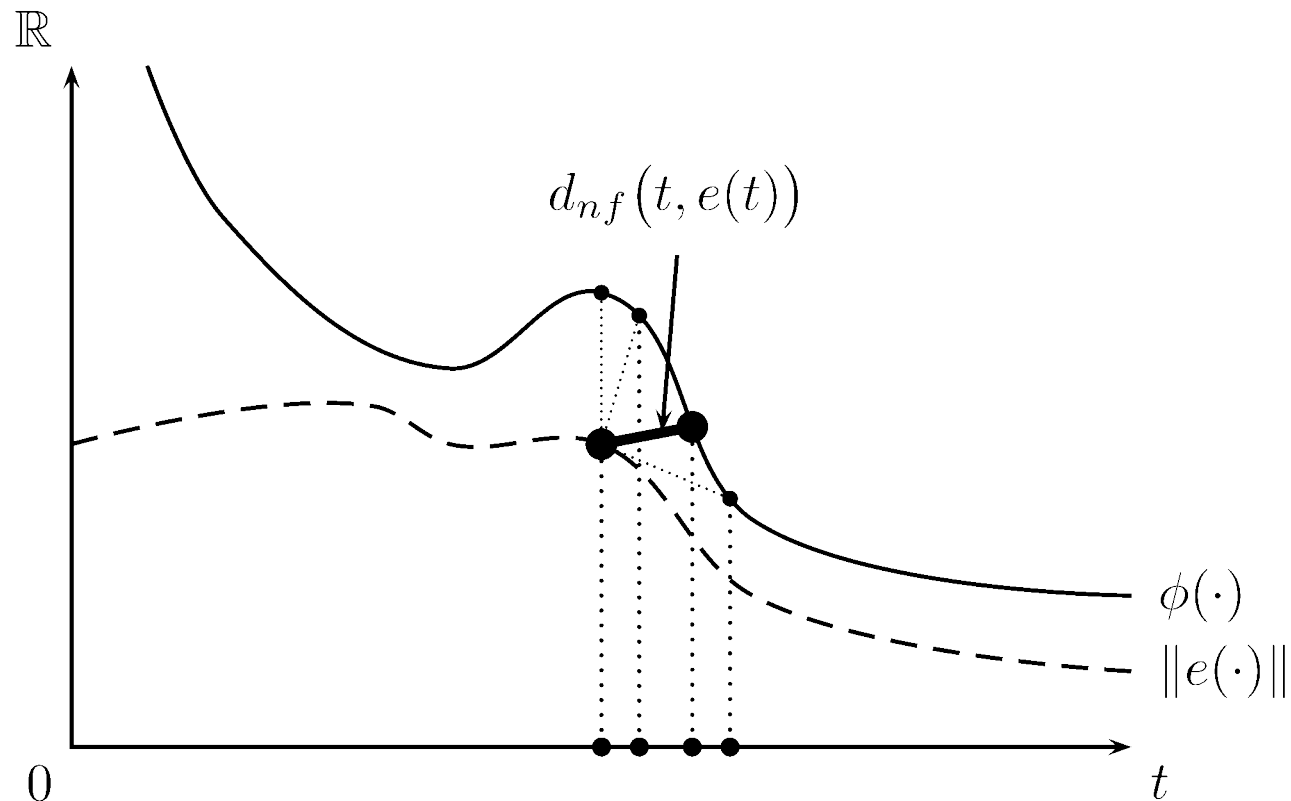
**Control input**  $u(t) = -k(t)e(t)$

$$k(t) = \frac{1}{d_f(t, e(t))} = K_{\mathcal{F}}(t, e(t))$$

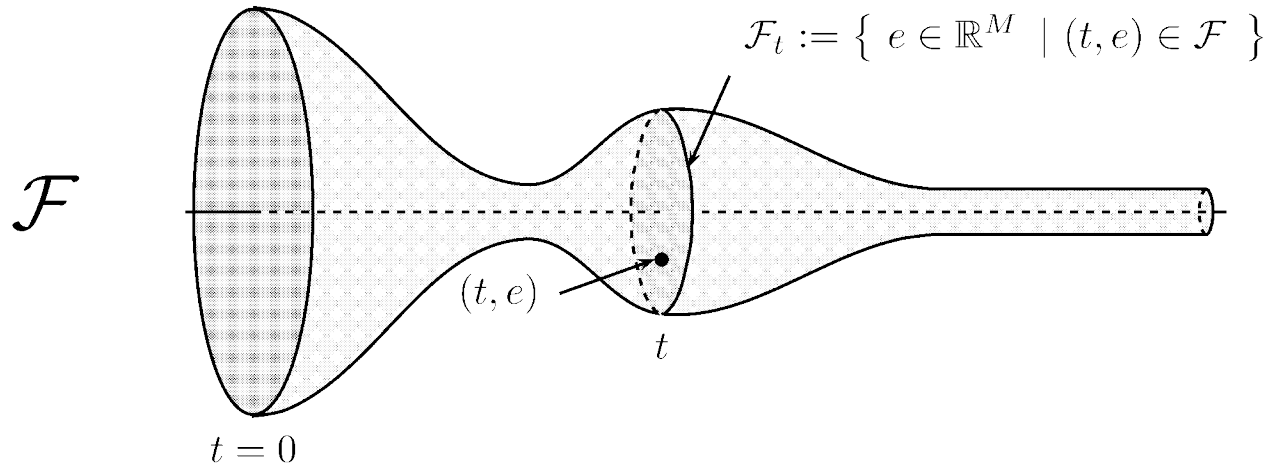


**Control input**  $u(t) = -k(t)e(t)$

$$k(t) = \frac{1}{d_{nf}(t, e(t))} = K_{\mathcal{F}}(t, e(t))$$



# Requisite properties of the gain function $K_{\mathcal{F}}$



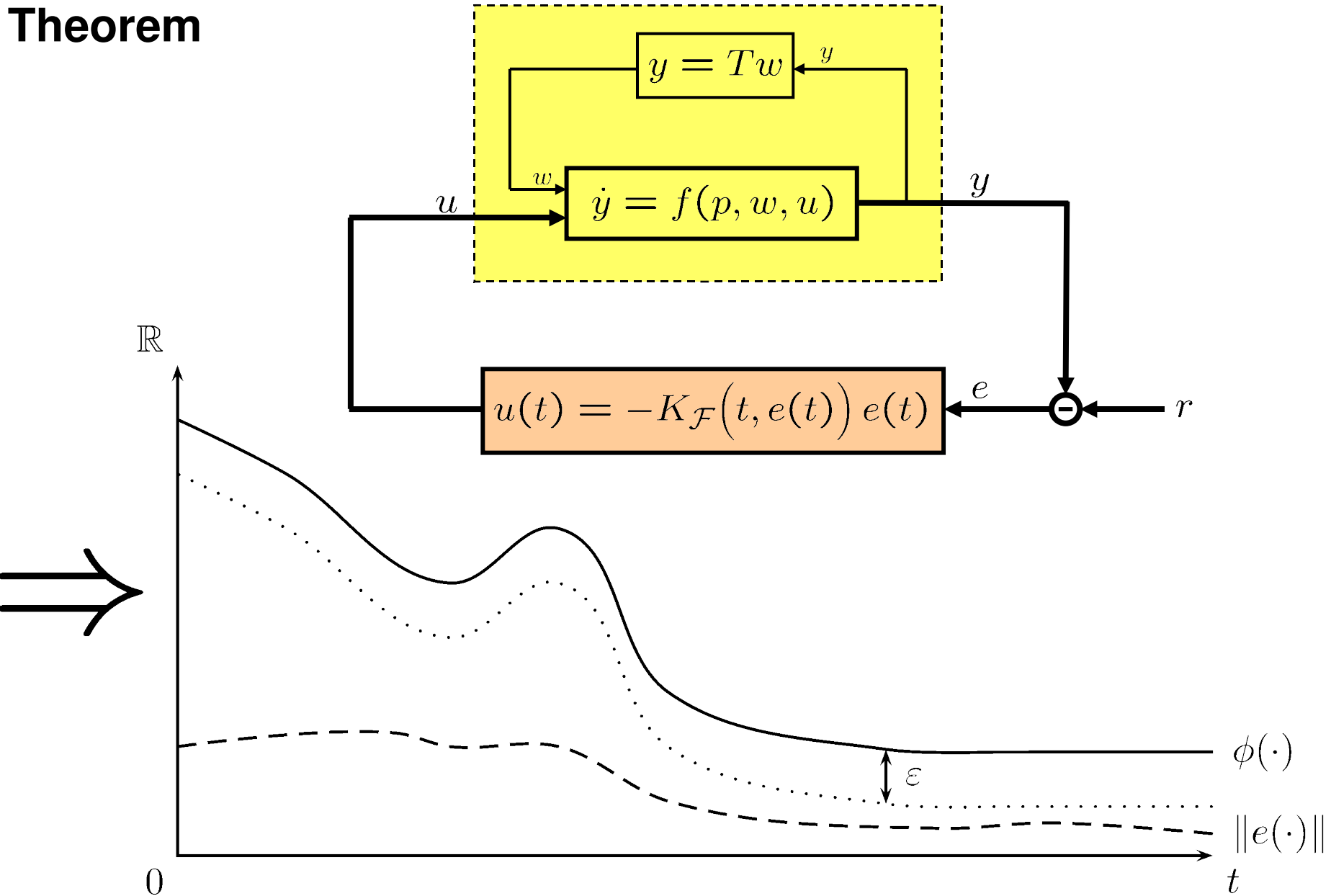
Property 1  $\forall K > 0 \exists \varepsilon > 0 \forall (t, e) \in \mathcal{F} :$

$$\text{dist}(e, \partial \mathcal{F}_t) \leq \varepsilon \Rightarrow K_{\mathcal{F}}(t, e) \geq K$$

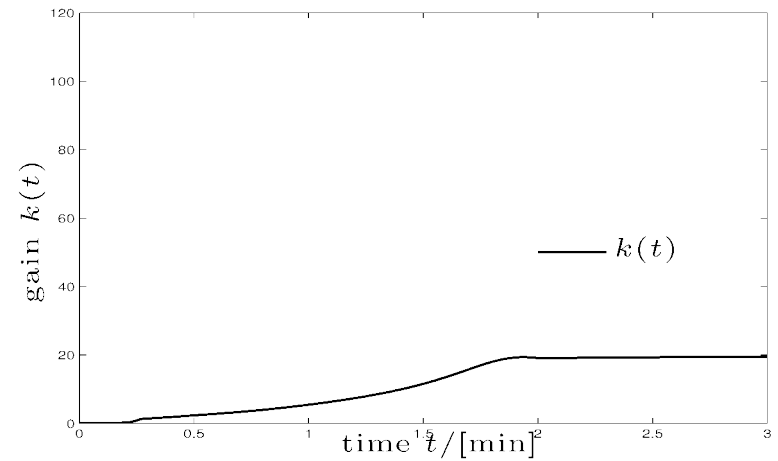
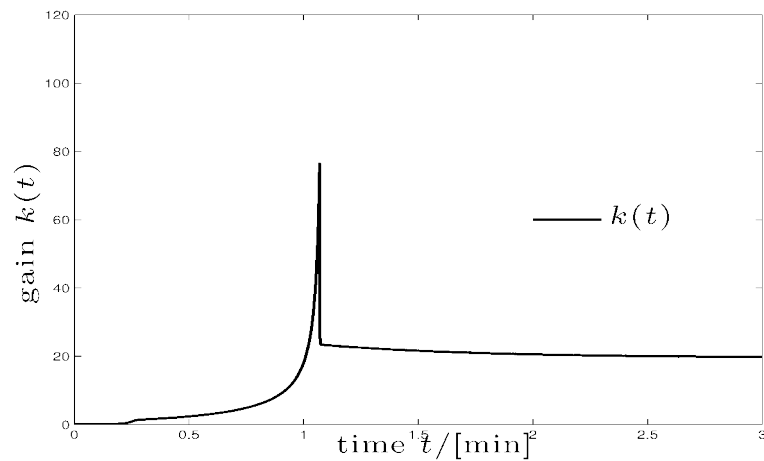
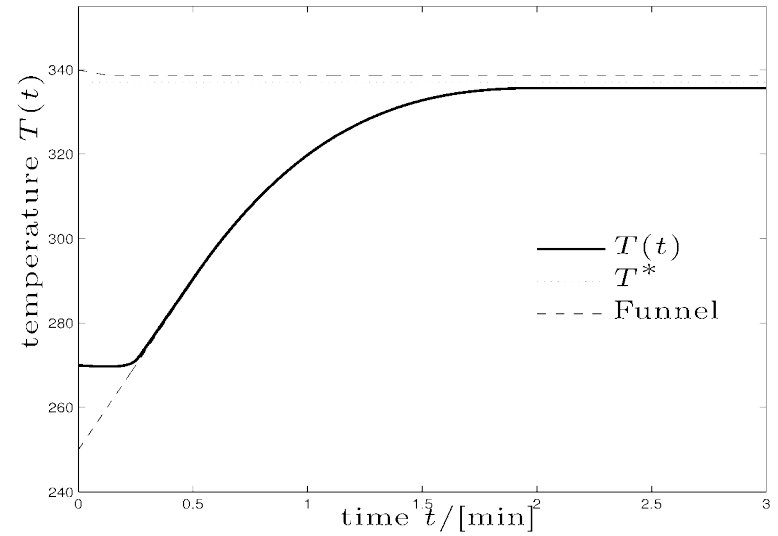
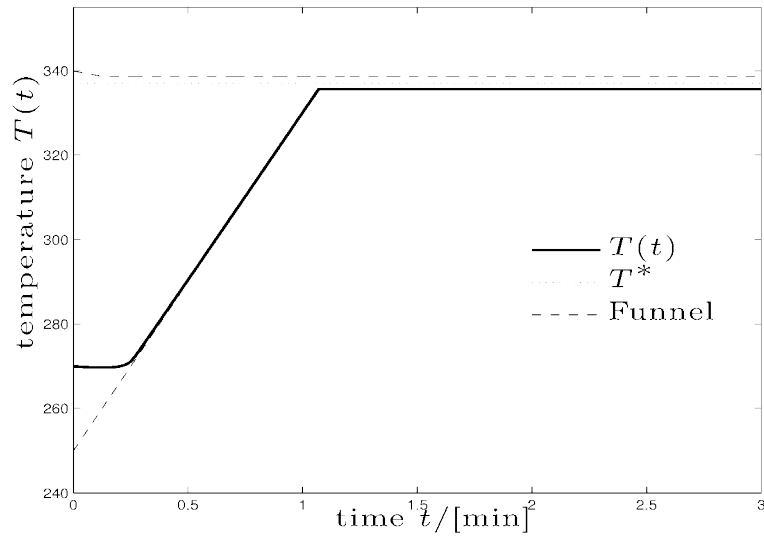
Property 2  $\forall \varepsilon > 0 \forall \delta > 0 \exists K > 0 \forall (t, e) \in \mathcal{F} :$

$$\left[ \text{dist}(e, \partial \mathcal{F}_t) \geq \varepsilon \text{ and } t \geq \delta \right] \Rightarrow K_{\mathcal{F}}(t, e) \leq K$$

## Theorem



# Applications: Bioreactor



## Theorem

