CDC-ECC'05

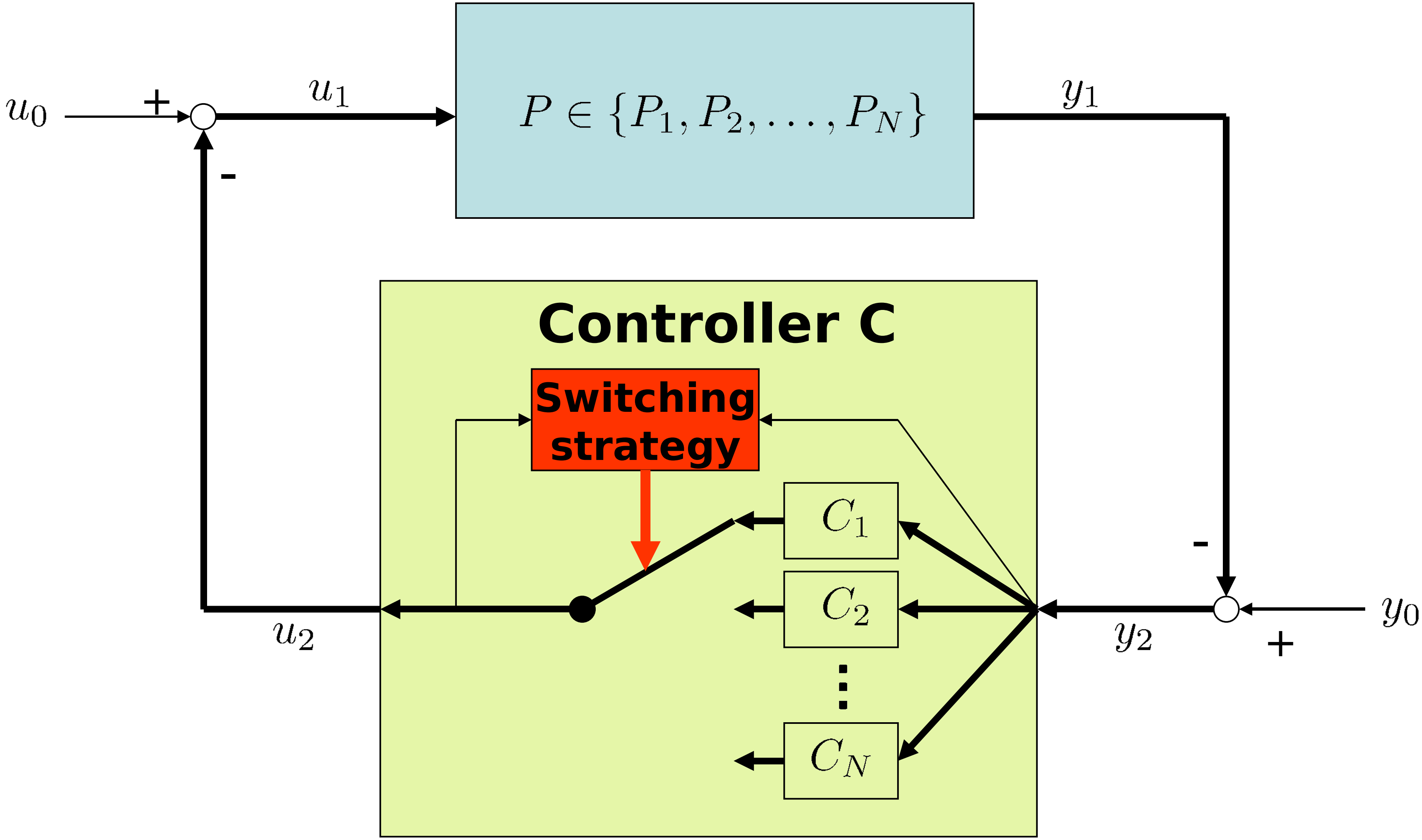
ℓ^p Gain Bounds for Switched Adaptive Controllers

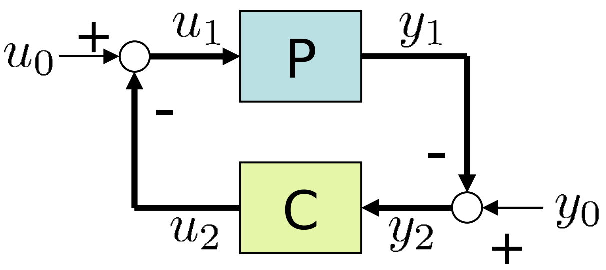
Mark French[a] and Stephan Trenn[b]

Sevilla, December 13th, 2005

[a] School of Electronics and Computer Science, University of Southampton, UK
[b] Institute of Mathematics, Technical University Ilmenau, DE

Switched control:



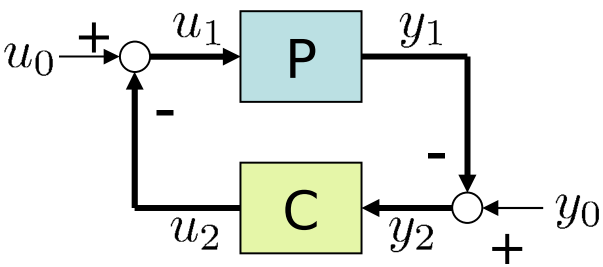


Aim:

$$\exists\, \gamma > 0 : \quad \left\| \begin{array}{c} u_2 \\ y_2 \end{array} \right\| \leq \gamma \left\| \begin{array}{c} u_0 \\ y_0 \end{array} \right\|$$

$$\overset{\text{Georgiou \& Smith `97}}{\Longrightarrow}$$ **robust stability**

In the following: $u_0, y_0 \in \ell^p =: V$



System class:

$$P = P_{p^*} \text{ for } p^* \in \{1, 2, \ldots, N\}$$

unknown

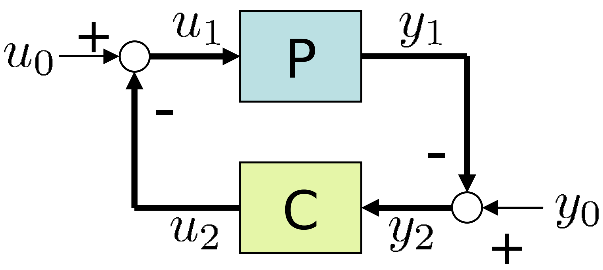
$$P_p : V_e \to V_e, \; u_1 \mapsto y_1$$

$$y_1(k) = \sum_{i=1}^{\sigma} a_{p,i} \, y_1(k-i) + b_p u_1(k-1)$$

$$=: L_p\Big(y_1\big|_{[k-\sigma,k-1]}, u_1(k-1)\Big)$$

known

CDC-ECC'05



Note that:

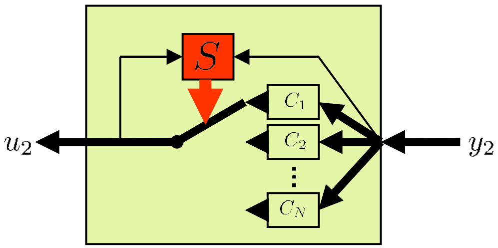
$$y_1(k) = L_p\Big(y_1\big|_{[k-\sigma,k-1]}, u_1(k-1)\Big) \quad \Rightarrow$$

$$y_2(k) = \hat{L}_p\Big(y_2\big|_{[k-\sigma,k-1]}, u_2(k-1), y_0\big|_{[k-\sigma,k]}, u_0(k-1)\Big)$$

Candidate controllers:

$$C_p : V_e \longrightarrow V_e,\ \ y_2 \longmapsto u_2$$

$$u_2(k) = -\frac{1}{b_p}\sum_{i=1}^{\sigma} a_p^i y_2(k-i+1)$$



Switching strategy *S*:

$$S: \ (u_2, y_2) \mapsto q, \text{ with}$$

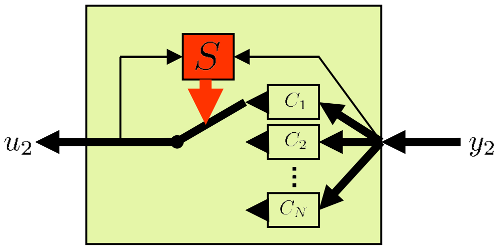
$$q(k) \in \{1, \ldots, N\} \text{ for all } k \in \mathbb{N},$$

and

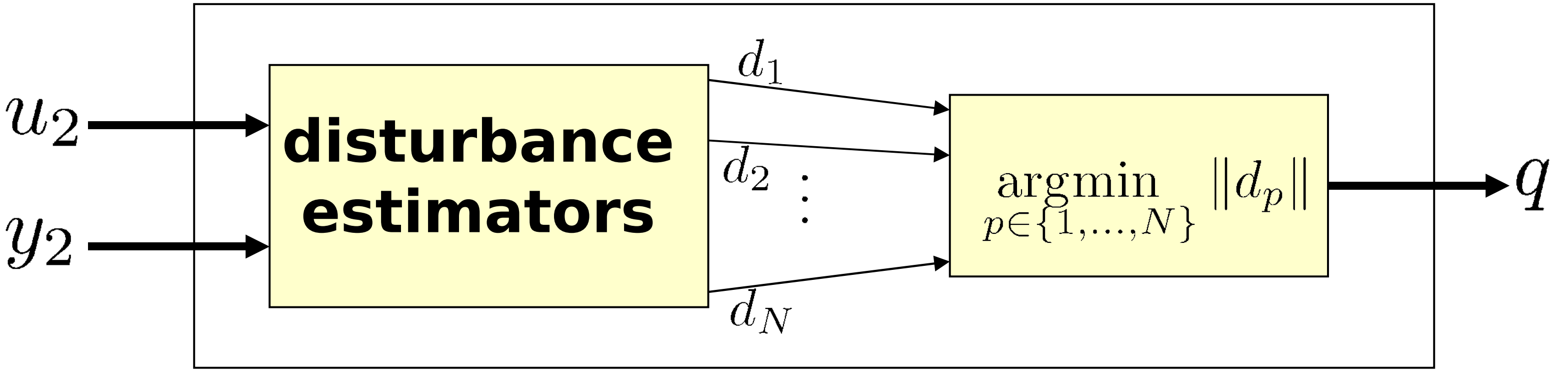
$$S\{u_2, y_2\}\big|_{[0,k]} = S\Big\{u_2\big|_{[0,k-1]}, y_2\big|_{[0,k]}\Big\}\Big|_{[0,k]}$$

Definition of switched controller:

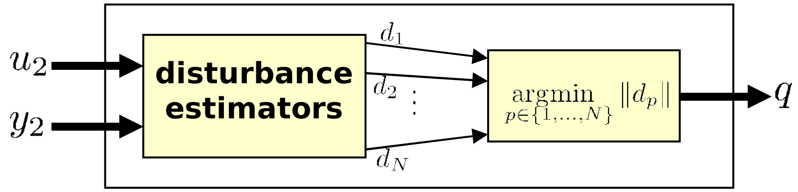
$$u_2(k) = C\{y_2\}(k) = C_{q(k)}\{y_2\}(k)$$



Structure of switching strategy:

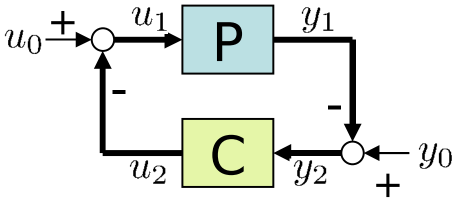


CDC-ECC'05



Assumptions on the disturbance estimators:

1. Estimators are causal.
2. Estimator for the real plant is bounded by real disturbance signal.
3. Estimators are consistent with the underlying candidate controllers.
4. Estimators are minimal.



Theorem:

Above Assumptions, then $\left\| \begin{matrix} u_2 \\ y_2 \end{matrix} \right\| \leq \gamma \left\| \begin{matrix} u_0 \\ y_0 \end{matrix} \right\|$

Remark:

- proof constructive, i.e. a γ can be calculated
- value of γ depends on parameters of real plant, on number of candidate plants and on distribution of pa

$$V = l^\infty \quad \Rightarrow \quad \gamma = \beta_\infty(\alpha_{p^*})^{N-1}$$

$$V = l^2 \quad \Rightarrow \quad \gamma = \sqrt{N}\beta_2(\alpha_{p^*})^{N-1}$$

Example: $P \in \{P_{-1}, P_0, P_1\}$, where for $a > 0$

$$P_p : y_1(k) = p\, a\, y_1(k-1) + u_1(k-1)$$

disturbance estimators: $\quad d_p(k) = \begin{pmatrix} u_0^{k-1} \\ y_0^k \\ y_0^{k-1} \end{pmatrix} \in \mathbb{R}^3$

$$\begin{pmatrix} u_0^{k-1} \\ y_0^k \\ y_0^{k-1} \end{pmatrix} = \operatorname{argmin} \left\{ \left\| \begin{matrix} u_0^{k-1} \\ y_0^k \\ y_0^{k-1} \end{matrix} \right\|_2 \;\middle|\; \begin{aligned} y_2(k) - y_0^k = p\, a\, &\left(y_2(k-1) - y_0^{k-1}\right) \\ &+ b\left(u_2(k-1) - u_0^{k-1}\right) \end{aligned} \right\}$$

Gain (upper bound) for Example:

$$\|y_2\|_2 \leq \gamma_{p^*} \|u_0, y_0\|_2 \text{ with}$$

$$\gamma_{\pm 1} = \sqrt{16a^8 + 128a^7 + 400a^6 + 57a^5 + 320a^4 + 32a^2}$$

$$\gamma_0 = \sqrt{117a^4 + 32a^2}$$