

Analogue Implementation of the Funnel Controller

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 - The funnel
 - The gain function
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Scope of funnel control



Aim

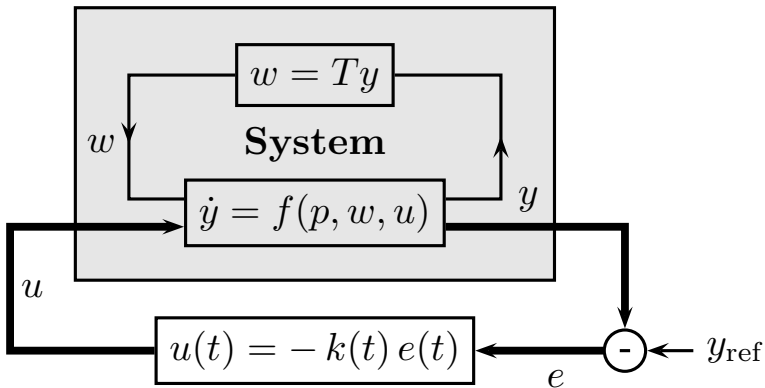
Tracking of a reference signal.

Scope of funnel control

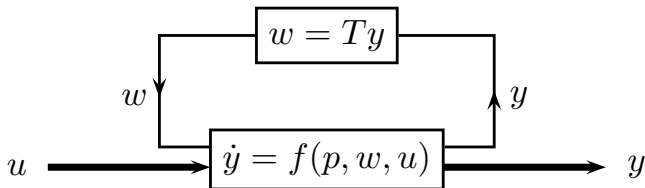


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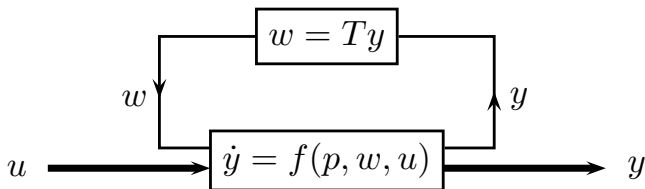
Tracking of a reference signal.



Properties of the system class

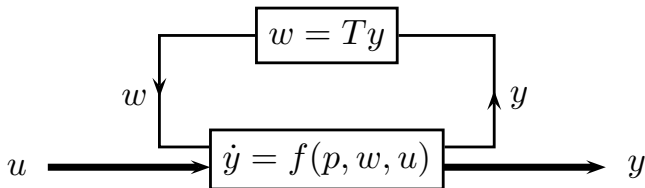


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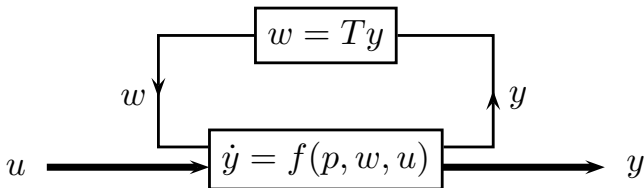
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Properties of the system class



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- bounded-input, bounded-output functional operators such as hysteresis and delays

Properties of the system class



- nonlinear functional differential equations
- bounded-input, bounded-output functional operators such as hysteresis and delays
- relative degree one and generalized high-frequency gain property

Control objectives



- **Practical asymptotic stability** of the error, i.e. for a given $\lambda > 0$

$$\exists T > 0 \quad \forall t \geq T : |e(t)| < \lambda.$$

- **Prescribed transient behaviour**, e.g. guaranteing an upper bound for the overshoot or an prescribed transient time.
- **Independence of system parameters**, i.e. the same controller works for all systems of the systems class.

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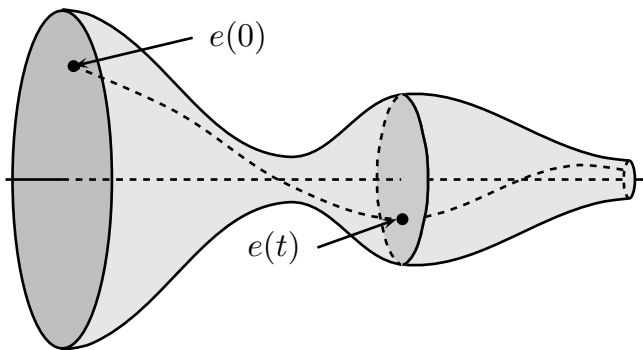
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Control objectives \Leftrightarrow prescribed funnel

The funnel $\mathcal{F} \subseteq \mathbb{R}_{\geq 0} \times \mathbb{R}^n$:



Architecture of the funnel controller



The control law:

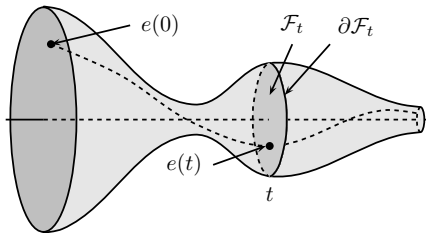
$$u(t) = -k(t) e(t)$$

The gain function

$$k(t) = K_{\mathcal{F}}(t, e(t))$$

$$K_{\mathcal{F}} : \mathcal{F} \rightarrow \mathbb{R}_{\geq 0}$$

Necessary properties of the gain function



Property A

$\forall K > 0 \exists \varepsilon > 0 \forall (t, e) \in \mathcal{F}$:

$$\text{dist}(e, \partial \mathcal{F}_t) \leq \varepsilon \Rightarrow K_{\mathcal{F}}(t, e) \geq K$$

Property B

$\forall \varepsilon > 0 \forall \delta > 0 \exists K > 0 \forall (t, e) \in \mathcal{F}$:

$$\text{dist}(e, \partial \mathcal{F}_t) \geq \varepsilon \text{ and } t \geq \delta \Rightarrow K_{\mathcal{F}}(t, e) \leq K$$

Theoretical results



Theorem

For any reference signal any consistent initial data, there **exists a solution** of the closed-loop initial-value problem.

Every solution can be extended to a maximal solution

$y : [-h, \omega) \rightarrow \mathbb{R}^n$ and every maximal solution has the following properties

- 1 $\omega = \infty$,
- 2 $t \mapsto k(t) = K_{\mathcal{F}}(t, e(t))$ is bounded on $\mathbb{R}_{\geq 0}$,
- 3 $\exists \varepsilon > 0 \forall t \in \mathbb{R}_{\geq 0}: \text{dist}(e(t), \partial F(t)) \geq \varepsilon$.

Proof in: Ilchmann, Ryan, Trenn (2005): *Tracking control: performance funnels and prescribed transient behaviour*

Further results



- **First funnel controller**

Ilchmann, Ryan, Sangwin (2002): *Tracking with prescribed transient behaviour*

- **Application to a model of chemical reactors**

Ilchmann, Trenn (2004): *Input constrained funnel control with applications to chemical reactor models*

- **Higher relative degree systems**

Ilchmann, Ryan, Townsend (2006): *Tracking with prescribed transient behaviour for nonlinear systems of known relative degree*

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Analogue Implementation



Now to Nagendra ...