

Analogue Implementation of the Funnel Controller

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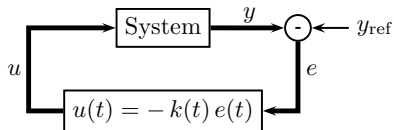


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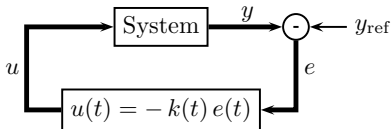
Scope of funnel control



Aim

Tracking of a reference signal.

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Tracking of a reference signal.

Properties of the system class

- nonlinear functional differential equations
- includes functional effects like hysteresis and delays
- high-gain stabilizable

Control objectives



- **Practical asymptotic stability** of the error, i.e. for a given $\lambda > 0$

$$\exists T > 0 \quad \forall t \geq T : |e(t)| < \lambda.$$

- **Prescribed transient behaviour**, e.g. guaranteing an upper bound for the overshoot or an prescribed transient time.
- **Independence of system parameters**, i.e. the same controller works for all systems of the systems class.

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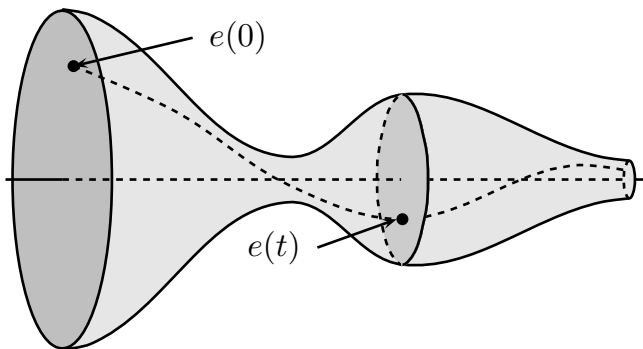
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Control objectives \Leftrightarrow Prescribed funnel

The funnel $\mathcal{F} \subseteq \mathbb{R}_{\geq 0} \times \mathbb{R}^n$:



Architecture of the funnel controller



The control law:

$$u(t) = -k(t) e(t)$$

The gain function

$$k(t) = K_{\mathcal{F}}(t, e(t))$$

$$K_{\mathcal{F}} : \mathcal{F} \rightarrow \mathbb{R}_{\geq 0}$$

Theoretical results



Necessary condition on the gain function $K_{\mathcal{F}}$

- 1 The closer the error to the funnel boundary, the larger the gain.
- 2 If the error is away from the funnel boundary then the gain is not unnecessarily large.

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Theorem

The funnel controller $u(t) = -K_{\mathcal{F}}(t, e(t)) e(t)$ achieves the control objectives, i.e. ensures that the errors evolves within the prespecified funnel independently of the system's parameters.

Proof in: Ilchmann, Ryan, Trenn (2005): *Tracking control: performance funnels and prescribed transient behaviour*

Further results



- **First funnel controller**

Ilchmann, Ryan, Sangwin (2002): *Tracking with prescribed transient behaviour*

- **Application to a model of chemical reactors**

Ilchmann, Trenn (2004): *Input constrained funnel control with applications to chemical reactor models*

- **Higher relative degree systems**

Ilchmann, Ryan, Townsend (2006): *Tracking with prescribed transient behaviour for nonlinear systems of known relative degree*

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Now to Nagendra ...