Analogue Implementation of the Funnel Controller

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   - Defintion of the funnel controller
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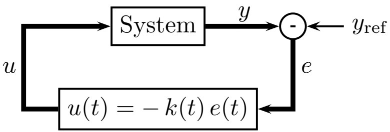
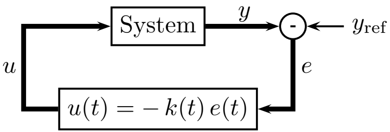
Scope of funnel control



Aim

Tracking of a reference signal.

Scope of funnel control



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Properties of the system class

- nonlinear functional differential equations
- includes functional effects like hysterises and delays
- high-gain stabilizable

Control objectives

- **Practical asymptotic stability** of the error, i.e. for a given $\lambda > 0$
$$\exists\, T > 0 \quad \forall\, t \geq T : \quad \big|e(t)\big| < \lambda.$$
- **Prescribed transient behaviour**, e.g. guaranteing an upper bound for the overshoot or an prescribed transient time.
- **Independence of system parameters**, i.e. the same controller works for all systems of the systems class.

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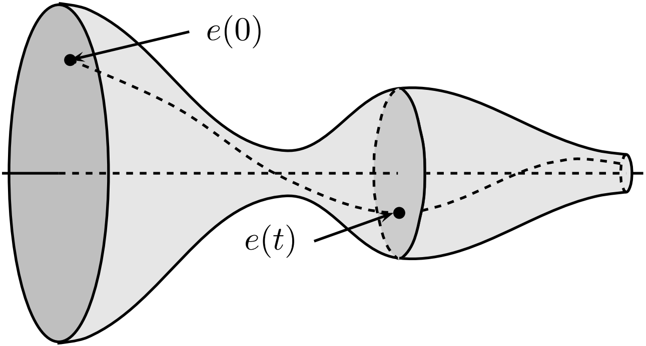
Control objectives $\Leftrightarrow$ Prescribed funnel

The funnel $\mathcal{F} \subseteq \mathbb{R}_{\geq 0} \times \mathbb{R}^n$:



Architecture of the funnel controller

The control law:

$$u(t) = -k(t)\, e(t)$$

The gain function

$$k(t) = K_{\mathcal{F}}\big(t, e(t)\big)$$

$$K_{\mathcal{F}} : \mathcal{F} \to \mathbb{R}_{\geq 0}$$

Theoretical results

Necessary condition on the gain function $K_{\mathcal{F}}$

1. The closer the error to the funnel boundary, the larger the gain.
2. If the error is away from the funnel boundary then the gain is not unnecessarily large.

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Theorem

The funnel controller $u(t) = -K_{\mathcal{F}}\big(t, e(t)\big)\, e(t)$ **achieves the control objectives**, *i.e. ensures that the errors evolves within the prespecified funnel independently of the system's parameters.*

Proof in: Ilchmann, Ryan, Trenn (2005): *Tracking control: performance funnels and prescribed transient behaviour*

Further results

- **First funnel controller**
  Ilchmann, Ryan, Sangwin (2002): *Tracking with prescribed transient behaviour*

- **Application to a model of chemical reactors**
  Ilchmann, Trenn (2004): *Input constrained funnel control with applications to chemical reactor models*

- **Higher relative degree systems**
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Now to Nagendra ...