Observer design for switched DAEs

... and what has Achim to do with it?

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Birthday colloquium Achim Ilchmann
12. February 2016, Elgersburg

TECHNISCHE UNIVERSITÄT
KAISERSLAUTERN

My first work with Achim: Funnel control, of course.

2004 CCA/ISIC/CACSD

Adaptive tracking within prescribed funnels

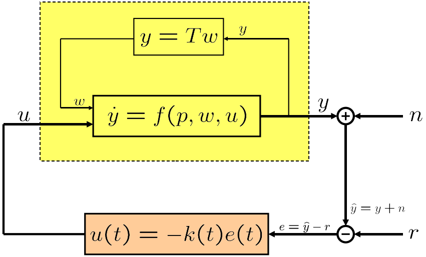
A. Ilchmann (TU Ilmenau, DE),
E.P. Ryan (Univ. of Bath, UK),
S. Trenn* (TU Ilmenau, DE)

September 3rd, 2004
Taipeh

* Stephan.Trenn@web.de

2004 CCA/ISIC/CACSD — *Adaptive tracking within prescribed funnels*

Aim: output tracking of nonlinear system by proportional error feedback



My first work without Achim: Switched systems

CDC-ECC'05

ℓ^p Gain Bounds for Switched Adaptive Controllers

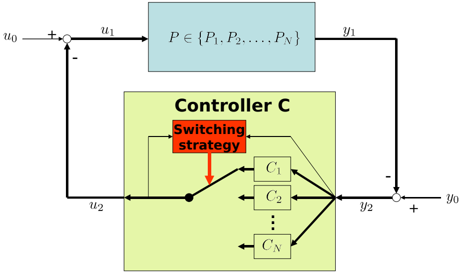
Mark French[a] and Stephan Trenn[b]

Sevilla, December 13th, 2005

[a] School of Electronics and Computer Science, University of Southampton, UK
[b] Institute of Mathematics, Technical University Ilmenau, DE

CDC-ECC'05

Switched control:



My first talk in Elgersburg, exactly 10 years ago

Anfangswertprobleme bei differential-algebraischen Gleichungen (DAEs)

Stephan Trenn

Institut für Mathematik, Technische Universität Ilmenau

Elgersburg, 13. Februar 2006

Differential-algebraische Gleichungen

$$\begin{aligned} &E(\cdot)\,\dot x = A(\cdot)\,x + B(\cdot)\,u \\ &E, A \in C^\omega(\mathbb{R};\mathbb{R}^{n\times n}),\ B \in C^\omega(\mathbb{R};\mathbb{R}^{n\times m}) \end{aligned} \tag{1}$$

Theorem (Campbell-Petzold 1983)

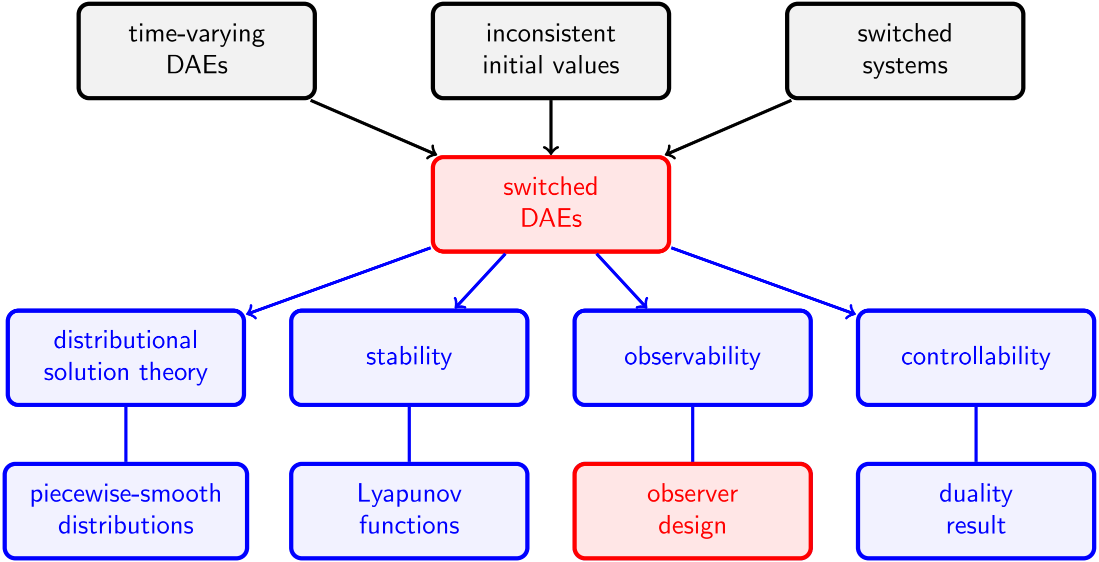
(1) *analytisch lösbar*

$\iff$

$\exists U \in C^\omega(\mathbb{R}, Gl_n),\ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = U^{-1}x$:

$$\begin{aligned} \dot x_1 &= A_1(\cdot)\,x_1 + B_1(\cdot)\,u, \\ N(\cdot)\,\dot x_2 &= x_2 + B_2(\cdot)\,u, \end{aligned} \qquad N = \begin{bmatrix} 0 & & * \\ & \ddots & \\ 0 & & 0 \end{bmatrix}$$

Switched DAEs

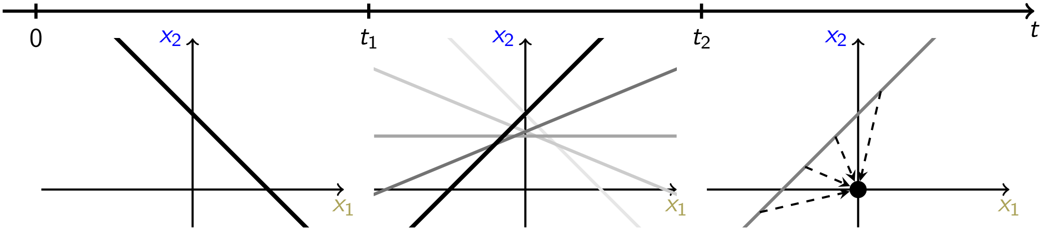


An example

$$\dot{x}_1 = 0 \quad \dot{x}_2 = 0 \quad \dot{x}_3 = 0 \quad \mathbf{y} = x_1 + x_2$$

$$\dot{x}_1 = -x_2 \quad \dot{x}_2 = x_1 \quad 0 = x_3 - x_2 \quad \mathbf{y} = 0$$

$$\dot{x}_1 = -x_2 \quad 0 = x_1 \quad \dot{x}_3 = 0 \quad \mathbf{y} = x_2$$



Can we reconstruct the state from the output?

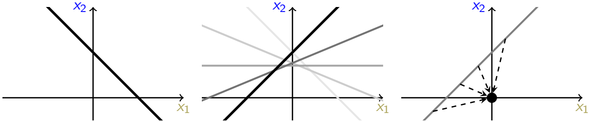
$$\mathbf{y}(t_1^-) = x_1(t_1^-) + x_2(t_1^-) = x_1(t_1^+) + x_2(t_1^+) \ \checkmark$$
$$x_3(t_1^-) = \ ?$$

$$t_2 - t_1 = \pi/4$$
$$x_2(t_2^-) = x_1(t_1^+) \ ?\checkmark$$
$$x_1(t_2^-) = -x_2(t_1^+) \ \checkmark$$

$$\mathbf{y}[t_2] = x_2[t_2] = x_1(t_2^-)\delta_{t_2} \ \checkmark$$
$$x_3(t_2^-) = x_2(t_2^-) \ ?\checkmark$$

$$x_1(t_2^+) = 0 \ \checkmark$$
$$x_2(t_2^+) = 0 \ \checkmark$$
$$x_3(t_2^+) = x_3(t_2^-) \ ?\checkmark$$

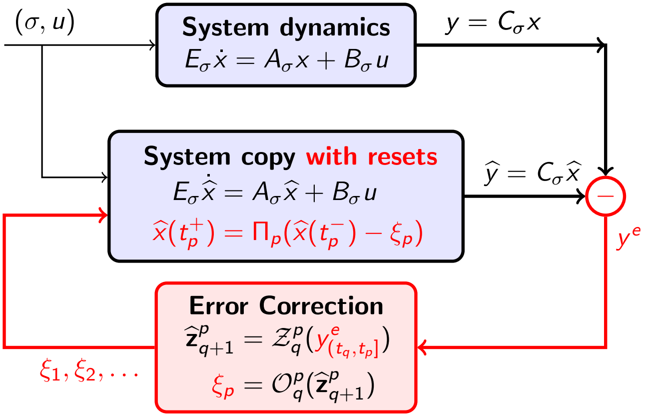
Discussion of example



- Non-observability of individual modes
- Several switches are necessary for reconstruction of states
- y on $[0, t_2]$ does not determine $x(0)$ but $x(t_2^+)$
  $\Rightarrow$ Observability vs. Determinability
- Partial knowledge of states have to be propagated in time and adequately combined with each other
- Algebraic constraints of states need to be utilized
- Dirac impulses contribute to reconstruction of state

EXCITING!

Overall observer structure



When does this work?

Persistent Determinability

How does it work?

Choice of ξ_p

Determinability

$$E_\sigma \dot{x} = A_\sigma x + B_\sigma u$$
$$y = C_\sigma x + D_\sigma u \qquad \text{(swDAE)}$$

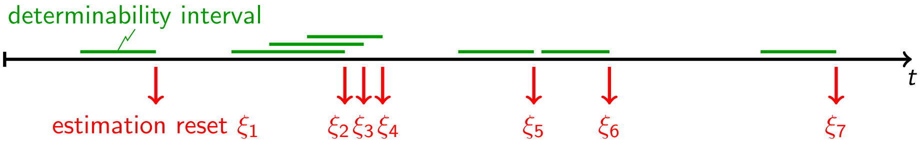
Definition (Determinability)

$(s,t]$ is a *determinability interval* of (swDAE)
$:\Leftrightarrow \forall (x,u,y), (\bar{x},\bar{u},\bar{y})$ solutions of (swDAE) on (s,∞) with $u=\bar{u}$:

$$y_{(s,t]} = \bar{y}_{(s,t]} \quad \Rightarrow \quad x_{(t,\infty)} = \bar{x}_{(t,\infty)}$$

Definition (Persistent Determinability)

(swDAE) is persistently determinable $:\Leftrightarrow \forall T \geq 0\ \exists$ determinability interval $(s,t]$ with $s \geq T$



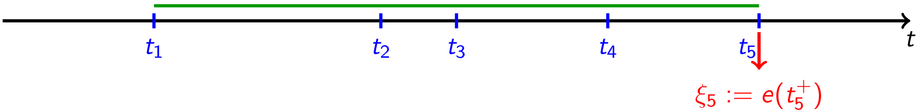
Error dynamics

Error dynamics (without resets)

$$\begin{aligned} e &:= \widehat{x} - x \\ y^e &:= \widehat{y} - y \end{aligned} \quad \text{are governed by} \quad \begin{aligned} E_\sigma \dot{e} &= A_\sigma e \\ y^e &= C_\sigma e \end{aligned}$$

Determinability assumption

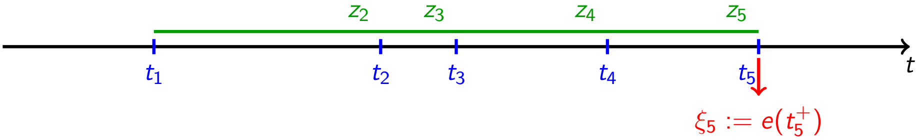
(swDAE) determinable on $(t_q, t_p]$ $\Rightarrow$ $e(t_p^+)$ reconstructable from $y^e_{(t_q,t_p]}$



Perfect Estimation

With $\xi_p := e(t_p^+)$ we have $\widehat{x}_{(t_p,\infty)} = x_{(t_p,\infty)}$

Recovering $\xi_p = e(t_p^+)$



Partial knowledge z_p

$$z_k = \begin{pmatrix} 0 \\ z_k^{\text{diff}} \\ z_k^{\text{imp}} \end{pmatrix} \begin{matrix} \leftarrow \quad \text{consistency information} \\ \leftarrow \quad \text{determined from } y_{(t_{k-1},t_k)} \\ \leftarrow \quad \text{determined from } y[t_k] \end{matrix}$$

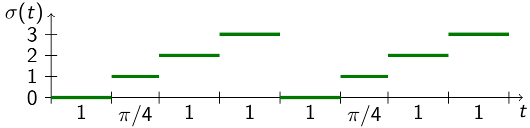
Combining partial knowledge

On determinability interval $(t_q, t_p]$:

$$\xi_p := e(t_p^+) = \Pi_p P_q^p \sum_{k=q+1}^{p} \left(\prod_{j=k+1}^{p} G_q^j \right) F_q^k z_k$$

for suitable matrices Π_p, P_q^p, G_q^j, F_q^k.

Simulations



$$(E_{4k+p}, A_{4k+p}, B_{4k+p}, C_{4k+p}) =$$

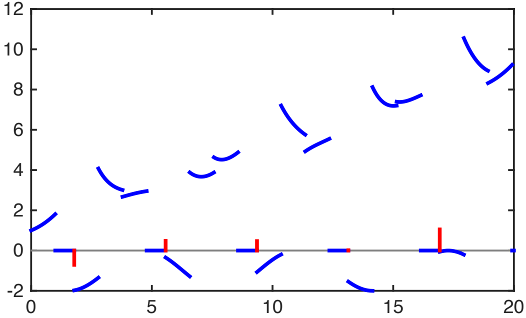
$$p=0: \quad \left(I, \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \right),$$

$$p=1: \quad \left(I, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & -1 & -1 & -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \right),$$

$$p=2: \quad \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \right),$$

$$p=3: \quad \left(I, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \right).$$

Input $u(t) = 1 + \sin t$ and initial value $x(0) = (1, 1.5, 2, 2.5)^\top$ yields the **output**:



State estimation erros (without Dirac impulses)