Observer Design for Detectable Switched DAE

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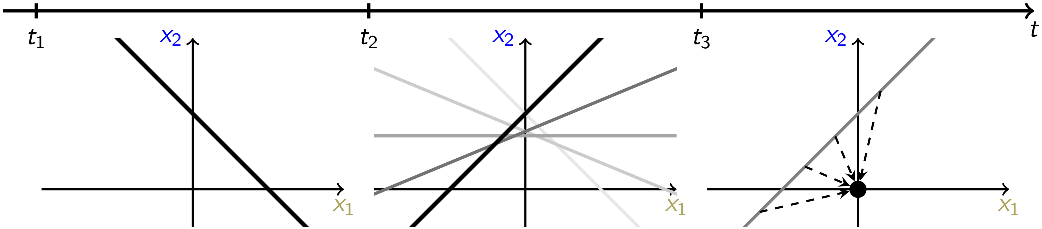
An example

$$\dot{x}_1 = 0, \quad \dot{x}_2 = 0, \quad \dot{x}_3 = 0, \quad \mathbf{y} = x_1 + x_2$$

$$\dot{x}_1 = -x_2, \quad \dot{x}_2 = x_1, \quad 0 = x_3 - x_2, \quad \mathbf{y} = 0$$

$$\dot{x}_1 = -x_2, \quad 0 = x_1, \quad \dot{x}_3 = 0, \quad \mathbf{y} = x_2$$



Can we reconstruct the state (at the final time) from the output?

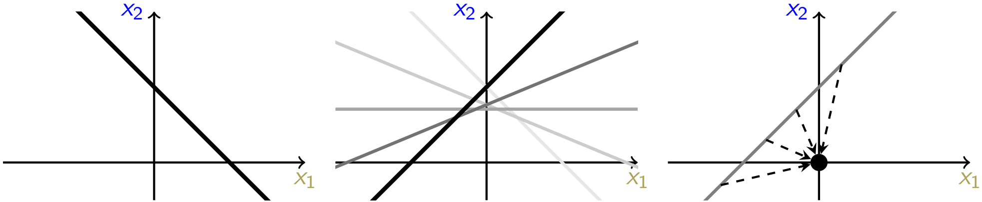
$$\mathbf{y}(t_2^-) = x_1(t_2^-) + x_2(t_2^-)$$
$$= x_1(t_2^+) + x_2(t_2^+) \checkmark$$
$$x_3(t_2^-) = \ ?$$

$$t_3 - t_2 = \pi/4$$
$$x_2(t_3^-) = x_1(t_2^+) \ ?\checkmark$$
$$x_1(t_3^-) = -x_2(t_2^+) \checkmark$$

$$\mathbf{y}[t_3] = x_2[t_3]$$
$$= x_1(t_3^-)\delta_{t_3} \checkmark$$
$$x_3(t_3^-) = x_2(t_3^-) \ ?\checkmark$$

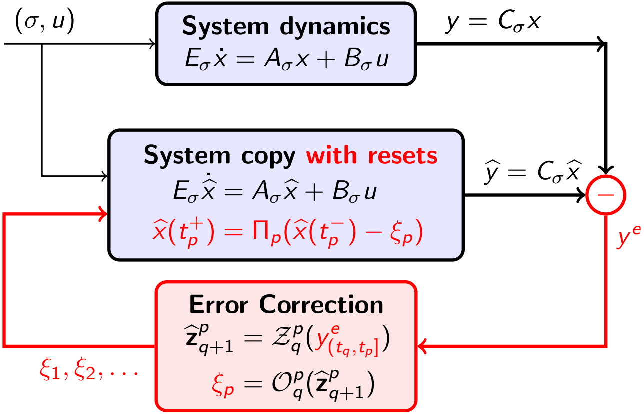
$$x_1(t_3^+) = 0 \checkmark$$
$$x_2(t_3^+) = 0 \checkmark$$
$$x_3(t_3^+) = x_3(t_3^-) \ ?\checkmark$$

Discussion of example



- Non-observability of individual modes
- Several switches are necessary for reconstruction of states
- y on $[t_1, t_3]$ does not determine $x(t_1^{\pm})$ but $x(t_3^{+})$
  $\Rightarrow$ Observability vs. Determinability
- Partial knowledge of states have to be propagated in time and adequately combined with each other
- Algebraic constraints of states need to be utilized
- Dirac impulses contribute to reconstruction of state

Observer structure from [Tanwani & T., Automatica 2016]



When does this work?

Persistent Determinability

How does it work?

Choice of ξ_p

Determinability vs. Detectability

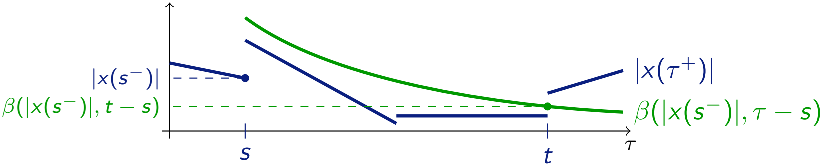
Known for classical observer design:

Determinability (i.e. observability) sufficient but not necessary for existence of observer!
$\longrightarrow$ detectability necessary and sufficient

Definition ($[s,t)$-Detectability, cf. [Tanwani & T., CDC'15])

(swDAE) is called $[s,t)$-*detectable* $:\Leftrightarrow$ $\forall(x, u \equiv 0, y \equiv 0)$ solutions on $[t,s)$:
$\exists$ $\mathcal{KL}$-function $\beta : \mathbb{R}^n \times [0,\infty) \to \mathbb{R}^n$ with $\beta(r, t-s) < r$ and

$$|x(\tau^+)| \leq \beta(|x(s^-)|, \tau - s) \quad \forall \tau \in [s,t)$$



Uniform interval-detectability

Theorem (Interval-detectability $\Rightarrow$ global detectability)

Consider (swDAE) with switching times $t_0 < t_1 < \ldots$ and assume $\exists\, p_0 < p_1 < \ldots$ such that

- *(swDAE) is $[t_{p_i}, t_{p_{i+1}})$-detectable with $\mathcal{KL}$-function $\beta_i \quad \forall i \in \mathbb{N}$*
- *$\exists$ uniform $\alpha \in (0,1)$: $\quad \beta_i(r, t_{p_{i+1}} - t_{p_i}) \leq \alpha r \quad \forall r > 0\ \forall i \in \mathbb{N}$*
- *$\exists$ uniform $M > 0$: $\quad \beta_i(r, 0) < M r \quad \forall r > 0\ \forall i \in \mathbb{N}$*

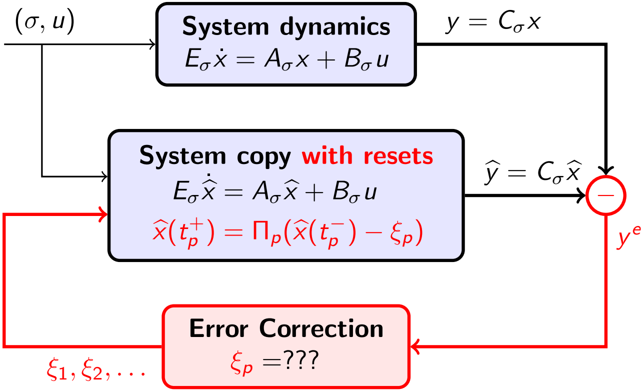
$\Longrightarrow \quad \forall\ (x, u, y), (\overline{x}, u, y)$ *solutions of (swDAE):*

$$x(t^{\pm}) - \overline{x}(t^{\pm}) \to 0 \quad \text{as} \quad t \to \infty$$

Remarks

- No upper bounds on length of detectable intervals
- Implicit dwell time condition via interval-detectability condition
- Result seems to be new even for switched ODEs

Observer structure



When does this work?

Uniform interval detectability

How does it work?

Choice of ξ_p

Novel Estimation Correction Necessary

$$\dot{x}_1 = 0,\quad \dot{x}_2 = 0,\quad \dot{x}_3 = 0,\quad \mathbf{y} = x_1 \qquad \text{on } [t_1, t_2)$$

$$\dot{x}_1 = x_2,\quad \dot{x}_2 = 0,\quad \dot{x}_3 = -x_3 + x_2,\quad \mathbf{y} = 0 \qquad \text{on } [t_2, t_3)$$

$$\dot{x}_1 = 0,\quad \dot{x}_2 = 0,\quad \dot{x}_3 = 0,\quad \mathbf{y} = x_1 \qquad \text{on } [t_3, t_4)$$

t_1 t_2 t_3 t_4 t

- System is detectable on $[t_1, t_4)$
- (x_1, x_2) is observable on $[t_1, t_4)$, x_3 is not observable
- Simple idea: Use known observer to update (x_1, x_2) at end of interval, keep x_3 unchanged

Simple approach not feasible

Estimation error for x_3 arbitrarily large!

Novel Estimation Correction Procedure

For each detectability interval do:

Step 1: Collect local observability data synchronously to systems dynamics

- Basically the same approach as in our previous observer design
- Main difference: local estimation needed at beginning of each subinterval

Step 2: Propagate back collected information for estimation correction at beginning of interval

- Based on recursively defined unobservability spaces, $k = q, q-1, \ldots, p$,

$$\mathcal{N}_k^q := \left\{ \, e(t_k^-) \;\middle|\; e \text{ solves error dynamics on } [t_k, t_q) \, \right\}$$

- Results in a “virtual” error correction ξ^{left} at the beginning of the interval

Step 3: Propagate ξ^{left} forward to obtain estimation correction at end of interval

- $\xi = e(t_q^-)$ where e solves error dynamics $\quad E_\sigma \dot{e} = A_\sigma e, \quad e(t_p^-) = \xi^{\text{left}}$
- No input and output storage needed